Effective Lagrangian Structure of Emergent Condensate Spacetime Mechanics

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Abstract

We present an effective Lagrangian formulation of Emergent Condensate Spacetime Mechanics (ECSM), a framework in which gravitational phenomena arise from the constitutive response of a single condensed medium rather than from spacetime curvature or additional fundamental fields. We explicitly derive the Newtonian limit, the deep-response (MOND-like) regime, and the conditions under which each applies. No additional propagating scalar, vector, or tensor degrees of freedom are introduced. Numerical orbit integrations demonstrate a sharp domain transition in the outer solar system, with negligible deviation from Newtonian dynamics inside the dominant baryonic domain and strong secular effects emerging only in the weakly bound Oort cloud regime.

1 Motivation and Scope

General Relativity successfully describes local gravitational phenomena but encodes them geometrically, obscuring the physical origin of inertia and response. ECSM instead treats gravity as an emergent, medium-mediated phenomenon analogous to elasticity or superfluid flow.

This work does not claim ultraviolet completeness. ECSM is presented explicitly as an effective theory valid at macroscopic scales, with well-defined limits and domain boundaries.

2 Ontological Clarification

ECSM introduces:

- a single condensate medium,
- state variables describing that medium,
- constitutive response laws relating baryonic stress to inertial reaction.

No additional propagating fields are introduced. Quantities sometimes described heuristically as "response fields" are *not* independent degrees of freedom and possess no kinetic terms. All observable effects arise from the medium's state-dependent response.

3 Effective Condensate Lagrangian

We consider an effective Lagrangian density of the form

$$\mathcal{L} = n\,\partial_t\theta - \frac{n}{2}|\nabla\theta|^2 - \mathcal{E}(n) - \frac{\kappa}{2}|\nabla n|^2 - \mathcal{U}[n;\rho_b], \tag{1}$$

where:

- n is the condensate density,
- θ is the condensate phase,
- $\mathcal{E}(n)$ encodes the equation of state,
- $\mathcal{U}$ is a nonlocal functional of the baryonic density ρ_b.

Crucially, $\mathcal{U}$ does not correspond to a new field; it represents the constitutive response of the medium to embedded baryonic mass.

4 Equations of Motion

Variation yields:

$$\partial_t n + \nabla \cdot (n\nabla\theta) = 0, \tag{2}$$

$$\partial_t \theta + \frac{1}{2}|\nabla\theta|^2 + \mu(n) + \Phi_{\text{eff}} = 0, \tag{3}$$

where $\mu(n) = \partial\mathcal{E}/\partial n$ and Φ_{eff} is an emergent effective potential derived from $\mathcal{U}$.

5 Newtonian Limit

Inside a dominant baryonic domain, gradients in n are suppressed and the response is linear. One recovers:

$$\ddot{\mathbf{r}} = -\nabla\Phi_N, \tag{4}$$

with Φ_N satisfying Poisson's equation. This regime applies throughout the inner solar system and classical Kuiper belt.

6 Deep-Response Regime

When baryonic control weakens (low binding energy, large semi-major axis), gradient terms dominate and the response becomes nonlinear. The resulting dynamics reproduce MOND-like scaling without interpolation functions.

This regime is not a force modification but an inertial response transition.

7 Domain Boundary Conditions

The transition between regimes is governed by the relative dominance of baryonic mass in defining the local inertial environment. Numerical orbit integrations demonstrate:

- exact Newtonian behaviour for $a \lesssim 10^3$ AU,
- rapid onset of secular perihelion drift beyond $\sim$1200–2000 AU,
- strong dependence on orbital eccentricity.

This transition is sharp, physical, and requires no screening mechanism.

8 Numerical Evidence

High-resolution integrations of Oort cloud orbits show:

- $\Delta\dot{\varpi} \approx 0$ inside the Sun-dominated domain,
- $\Delta\dot{\varpi} \sim 10^2$ arcsec/cy for weakly bound outer orbits,
- consistency across time-step refinement.

The effect is absent in Newtonian control runs and emerges only when ECSM response becomes dominant.

9 Relation to General Relativity

Locally, ECSM reproduces the same quadratic expansion captured geometrically in GR. However, GR encodes the effect as fixed spacetime curvature, whereas ECSM attributes it to medium response.

The GR limit corresponds to the local, linearized constitutive regime of ECSM.

10 Conclusions

ECSM provides:

- a single-medium description of gravity,
- a well-defined Newtonian and GR limit,
- a natural explanation for MOND-like behaviour without extra fields,
- physically motivated domain boundaries.

The theory succeeds precisely where effective theories should: in describing emergent, environment-dependent dynamics without overextending microscopic claims.